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Formula to evaluate the area of irregular convex quadrilaterals

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Abstract— This research provides a general formula to calculate the area of irregular convex quadrilaterals.

$$Area = \frac{[P \sin \Theta \times (S - R \cos \varphi) + R \sin \varphi \times (S - P \cos\Theta)]}{2}$$

Index Terms— Area of Quadrilateral, Convex quadrilaterals, Irregular quadrilaterals

INTRODUCTION:

The following research was conducted in order to present a general formula to evaluate the area of an irregular convex quadrilateral. The Formula works for all kinds of convex quadrilaterals. The area for a quadrilateral below can be given by the following expression where Θ is the angle between sides' p and q, and φ is the angle between sides r and s.

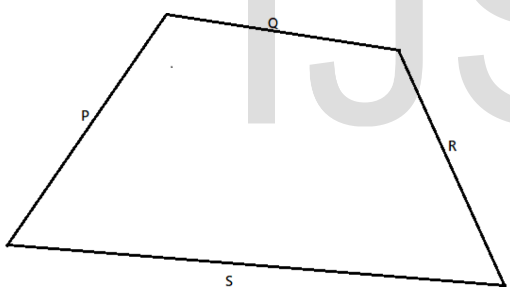

fig .(1.1)

$$\frac{[P \sin \Theta \times (S - R \cos \varphi) + R \sin \varphi \times (S - P \cos\Theta)]}{2}$$

SCOPE:

The following formula would be very helpful in geometry as it forms a very basic and general formula for irregular convex quadrilaterals.

OBJECTIVE:

The objective behind making this formula was to generalize all the formulae of area of convex quadrilaterals and to give a formula such that it can evaluate the area of any random irregular convex quadrilateral. Using this formula any other regular convex quadrilateral's area could be derived.

MAIN BODY:

First we would like to discuss about what an irregular convex quadrilateral means. Here, the word convex tells that all angles inside the quadrilateral are acute angles. All sides and angles are irregular, which means that the quadrilateral has all four sides and angles of unique lengths and degrees respectively without any special pattern. No two opposite sides are parallel. From these given conditions how can we design a formula to provide the area of the quadrilateral?

The formula can be derived via the following method:

Consider the longest side of the quadrilateral as our base and construct two perpendiculars on the base such that each perpendicular is joined with the upper two vertices A and C.

Let the lengths of sides **IJ = S**, **AI = p**, **CJ = R** and **AC=Q**

- Construction work: construct 2 perpendicular lines with respect to the base as given in the diagram below

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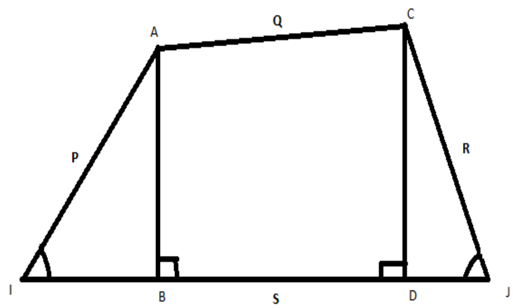

fig.(1.2)

Now we will use a simple idea of adding three simple shapes from fig. (1.2)

Area of AIB + Area of ABDC + Area of CDJ = Area of AIJC

Area of AIB (triangle) = $\frac{P^2 \cos\Theta \sin\Theta}{2}$

Area of CDJ (triangle) = $\frac{R^2 \cos\varphi \sin\varphi}{2}$

Area of ABDC (trapezium) = $\frac{(P\sin\Theta + R\sin\varphi)(S - (R\cos\varphi + P\cos\Theta))}{2}$

Now area of the quadrilateral could be given by the following expression

$$area = \frac{P^2 \cos\Theta \sin\Theta + R^2 \cos\varphi \sin\varphi + (P\sin\Theta + R\sin\varphi)(S - (R\cos\varphi + P\cos\Theta))}{2}$$

Let $P^2 \cos\Theta \sin\Theta + R^2 \cos\varphi \sin\varphi = \beta$

$$area = \frac{\beta + (P\sin\Theta + R\sin\varphi)(S - (R\cos\varphi + P\cos\Theta))}{2}$$

$$area = \frac{\beta + (S(P\,sin\,\Theta + R\,sin\,\varphi) - (R\cos\varphi + P\cos\Theta)(P\,sin\,\Theta + R\,sin\,\varphi))}{2}$$

$$area = \frac{\beta + (S(P\,sin\,\Theta + R\,sin\,\varphi) - (PR(sin\,\Theta\cos\varphi + sin\,\varphi\cos\Theta) + \beta))}{2}$$

$$area = \frac{SP\,sin\,\Theta + SR\,sin\,\varphi - PR\,sin\,\Theta\,cos\,\varphi - PR\,sin\,\varphi\,cos\,\Theta}{2}$$

$$area = \frac{P\,sin\,\Theta\,(S - R\cos\varphi) + R\,sin\,\varphi(S - P\cos\Theta)}{2}$$

This is our main equation from which we can evaluate the area of an irregular convex quadrilateral just by knowing the two base angles and length of the base and two sides adjacent to it.

It can be simplified by writing it in another form;

$AREA = 0.5\,((AB \times ID) + (CD \times BJ))$

If we join the sides AD and BC we can clearly see that the area of the quadrilateral is the sum of the areas of ΔAID and ΔCBJ fig. (1.3)

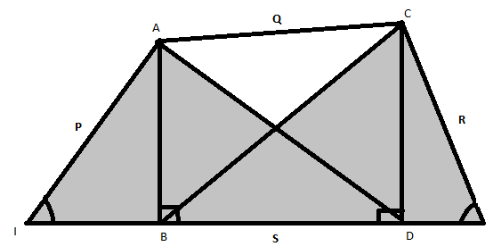

fig. (1.3)

CONCLUSION:

The overall research gives us an important result which is

The area of an irregular convex AIJC with two perpendiculars AB and CD on IJ can be given by the sum of areas of triangle AID and CBJ. The mathematical formulation of it is given by

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$$Area\ of\ AIJC = \frac{P\sin\Theta\,(S - R\cos\varphi) + R\sin\varphi(S - P\cos\Theta)}{2}$$

$$Area\ of\ AIJC = Area\ of\Delta\ AID + Area\ of\Delta CBJ$$

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